Abstract for **Non Ideal Measurements**
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We have previously objected to "modal interpretations" of quantum theory by showing that these interpretations do not solve the measurement problem since they do not assign outcomes to non_ideal measurements. Bub and Healey replied to our objection by offering alternative accounts of non_ideal measurements. In this paper we argue first, that our account of non_ideal measurements is correct and second, that even if it is not it is very likely that interactions which we called non_ideal measurements actually occur and that such interactions possess outcomes. A defense of the modal interpretation would either have to assign outcomes to these interactions, show that they don't have outcomes, or show that in fact they do not occur in nature. Bub and Healey show none of these.

**Key words:** Foundations of Quantum Theory, The Measurement Problem, The Modal Interpretation, Non_ideal measurements.
Non_Ideal Measurements

Some time ago, we raised a number of rather serious objections to certain so-called "modal" interpretations of quantum theory (Albert and Loewer, 1990, 1991). Andrew Elby (1993) recently developed one of these objections (and added some of his own), and Richard Healey (1993) and Jeffrey Bub (1993) have recently published responses to us and Elby. It is the purpose of this note to explain why we think that their responses miss the point of our original objection.

Since Elby's, Bub's, and Healey's papers contain excellent descriptions both of modal theories and of our objection to them, only the briefest review of these matters will be necessary here. According to standard quantum theory, an ideal non_disturbing measurement of a discrete_valued observable Q of system S by an observable A of system M (e.g., A may be the position of a pointer on a measuring device) is an interaction governed by a Hamiltonian which satisfies the following condition: If the initial state of S+M is $|Q=q_i\rangle|A=r\rangle$ ($|A=r\rangle$ is the ready_to_measure state of M), then the post_measurement state of S+M (the state immediately after the measurement is completed) is $|Q=q_i\rangle|A=a_i\rangle$. It follows that, if the measurement is governed by a linear equation (e.g., Schroedinger's equation) and if the initial state of S+M is $\sum c_k |Q=q_k\rangle|A=r\rangle$, then its post_measurement state is $\sum c_k |Q=q_k\rangle|A=a_k\rangle$. Further, if, as the orthodox interpretation of quantum theory maintains, an observable of a system has a value when and only when the state of that system is an eigenstate of that observable, then, since this state is not an eigenstate of either Q or A, it follows that neither observable has a value in this state. And that, of course, is a problem, since (once again) A here denotes the position of a macroscopic pointer, and our experience seems to afford us very good reasons to believe that there are invariably determinate matters of fact about the positions of pointers like that.

Modal interpretations attempt to solve this problem by denying that an observable for a system has a value only when the state of the system is an eigenstate of the observable. In Healey's words they "break the eigenstate_eigenvalue link." Bub's and
Healey's versions of modal interpretations go about this in rather different ways. Our objection was directed at Healey's approach, not Bub's, but both argue that the objection is not decisive against Healey's account. We will be mainly concerned with these arguments but we will have a few words to say later about Bub's interpretation.

Healey's modal interpretation contains a recipe which predicts which observables possess definite values and then assigns probabilities to the values of these observables. Here is how it works: According to the polar_decomposition theorem, every quantum state of a system like $S+M$ can be represented in the bi_orthonormal form

$$\sum c_k |Q=q_k> |A=a_k>,$$

where the products $|Q=q_i> |A=a_j>$ for all $i,j$ form a complete basis for $S+M$. Moreover, the theorem entails that, as long as not all the $c_k$ are equal, this representation is unique. Healey's interpretation stipulates that, if (1) is the unique bi_orthonormal state of $S+M$, then the observables $Q$ and $A$ possess determinate values even though (1) is not an eigenstate of either of them.

And that will entail (and this is the punch line) that it will invariably be the case, at the conclusions of ideal non_disturbing measurement interactions, that there will be a determinate matter of fact about the value of the "measured observable" and (even more importantly) that there will be a determinate matter of fact about whatever counts as the "position of the pointer" on the "measuring device." And that, of course, will solve the problem described above.

The trouble, though, is that the problem described above is only a very special case of the problem at the foundations of quantum mechanics that actually needs to be solved. That latter problem (which has come to be called, a little misleadingly, "the measurement problem") is not merely to account for the determinateness of the positions of pointers at the conclusions of ideal non_disturbing measurement_interactions, but to account for the determinateness of whatever actually happens to be determinate, under whatever circumstances that
determinateness actually happens to arise. Manifestly, no interpretation of quantum theory which offers anything less than an account of all that (whatever the problem of producing such an account is called) will do.

Now, what we pointed out was that, while Healey's prescription does assign a determinate position to the pointer in (1), which arises at the conclusion of an ideal non_disturbing measurement_interaction, it fails to assign determinate positions to pointers in certain other states, which arise at the conclusion of certain other processes. Consider, for example, an interaction between M and S which satisfies the following condition:

\[ |Q=q_i>|A=r> \rightarrow \text{SUM } c_{ij}|Q=q_i>|A=a_j> \]  

As before, the observable A represents the pointer position. This interaction creates problems for Healey's interpretation, since the post_measurement state of M+S will have the following bi_orthonormal form:

\[ \text{SUM } c_k|Q^*=q_k>|A^*=a_k> \]  

where Q* are A* are observables that fail to commute with Q and A. On Healey's prescription, when S+M is in state (3), Q* and A* possess values, but Q and A do not. Further, A* need not even be "close" to A, even if \[ \text{SUM } c_{ij}|Q=q_i>|A=a_j> \] is close to (1). So, if interactions like (2) are really possible, then there are interactions in which measuring devices end up in states like (3) in which, according to Healey's interpretation, pointers fail to have positions, or even approximately have positions. The question for the modal interpretation then is whether interactions like (2) actually occur. We argued previously that they do occur and that, in fact, non_ideal or not completely accurate measurements are modeled by (2). We will first defend this claim.

In setting up an ideal measurement, the Hamiltonian governing the interaction between M and S must induce a perfect correlation between Q and S. Of course, it will prove enormously difficult to set up a process governed by such a Hamiltonian. A near miss will instead almost certainly result in an interaction which is modeled by (2). The reason is that in the vicinity of a Hamiltonian which induces a perfect correlation almost all Hamiltonians yield interactions
modeled by (2). Further, interactions which satisfy (2) provide a natural model of inaccurate such measurements since, given the usual probabilistic interpretation of the amplitudes that occur on the right-hand side of (2), we can calculate the degree of correlation between Q and A. When the $c_{ij}$ for $i \neq j$ are zero the correlation is perfect; when the $c_{ij}$ are small, the correlation is high. For example, in a standard spin measurement in which the state of a particle is separated in a magnetic field with, say, a spin_up component going in one direction and a spin_down component going in a different direction, there will always be some amplitude of finding a spin_up (down) particle in the spin_down (up) region. Bub gives a curious argument against (2) as a model of inaccurate measurements. He says

"Now, it seems to me that this way of representing non_ideal measurements begs the question, by presupposing that the $c_{ij}$ can be interpreted as specifying epistemic probabilities that the observable Q actually has the value $q_i$ and the observable A actually has the value $a_j$ for all i and j."

But it is Bub who is begging the question, not us. It is question begging to argue that (2) does not correctly describe inaccurate measurements because the modal interpretation does not assign definite pointer positions to (3). It seems perfectly reasonable to assume that, at the conclusion of the measurement described by (2), the pointer has a definite position and that the probabilities of its various positions are given by the usual quantum mechanical rules. It is the task of an adequate interpretation to make sure that it has a definite position or, failing that, to explain why it just seems to have a definite position. And, there are other solutions to the measurement problem_Bohm's theory, the GRW collapse theory, and, for that matter Bub's own version of the modal interpretation in which interactions conforming to (2) yield states in which the pointer has a position with the appropriate probabilities."

Bub and Healey propose a couple of alternative accounts of non_ideal measurement. According to one (proposed by both Bub and Healey) a non_ideal measurement is a measurement by a device which correlates values of A with values of Q only a certain
percentage, say 90%, of the time. Another proposal (suggested by Healey) is that M doesn't measure Q but some observable that is close to Q. Both of these accounts actually identify non_ideal measurements of Q with ideal measurements; in one case with a type of interaction which (depending on initial conditions) is sometimes an ideal measurement of Q and sometimes an ideal measurement of other observables and, in the other case, with an ideal measurement of an observable close to Q. Neither of these accounts of non_ideal measurements are problematic for the modal interpretation since on either of them the bi_orthonormal form of the post_measurement state is expressed in terms of eigenstates of A.

We can grant that some non_ideal measurements are described by these proposals. We can even grant, contrary to what we think, that inaccurate measurements are not modeled by (2). These concessions do not save save Healey's modal interpretation from our objection. This is because our objection is that interactions conforming to (2)_whether or not they are called "inaccurate measurements"_ result in states like (3) in which on Healey's interpretation pointers fail to possess positions. To answer our objection, the defender of Healey's version of the modal interpretation must show either that there are no actually occurring physical processes which are described by (2) or that, despite what we may think, a pointer in (3) actually would not possess a definite position. Bub and Healey come nowhere near showing either of these.

It should be clear that the burden is on Healey and Bub to show that states like (3) do not arise in circumstances where we know, or think we know, that Q has a definite value. This will be enormously difficult since, as we mentioned previously, in the neighborhood of every Hamiltonian that characterizes an ideal measurement there are Hamiltonians that characterize evolutions like (2). In fact, on natural measures the measure of the set of Hamiltonians which correspond to ideal measurements is 0. Since, on the accounts of Bub and Healey, non_ideal measurements of Q are really ideal measurements of some other observable or are ideal measurements of Q some of the time and some other observables the rest of the time, the measure of these
interactions is also 0. Now, it is possible that the initial state of the universe and the Hamiltonian which actually characterizes the evolution of that state never results in a state like (3) (in which pointer positions, and other observables that we are sure always possess values, are not assigned values by Healey's modal interpretation). But, it should be clear that this is an enormously unlikely possibility.

As we previously mentioned, Bub's version of the modal interpretation does not have a special problem with non-ideal measurements. Unlike Healey, his account doesn't include a law that determines which observables possess definite values. Instead, Bub simply stipulates that pointer observables and measured observables, whatever they may be, possess definite values. Thus, in state (2), A and Q have values because they are, by stipulation, the pointer and measured observables. Bub shows that this stipulation is consistent with the quantum mechanical formalism (specifically it doesn't run into trouble with Gleason's theorem). This "solution" of the measurement problem can hardly be thought to be satisfactory. The trouble is that which observables count as pointer and measured observables is imported from outside of the theory. That quantum mechanics (once the eigenstate-eigenvalue link is dropped) is consistent with those observables which we think have values actually having values is reassuring, but we expect the states posited by an adequate fundamental theory to tell us exactly which observables possess values. Bub's modal interpretation fails to do this and so is incomplete. To complete it one would need to add explicit rules specifying which observables possess values and the probabilities of those values. This is what Healey's account does. Unfortunately for that account, as we have argued in this paper, the rules do not assign values to observables that we believe do have values.

We want to conclude with a brief diagnosis of why Bub and Healey failed to appreciate the dialectic of our argument. It had been quite common until fairly recently to attempt to understand quantum mechanics instrumentally; that is, as a theory whose aim is not to describe real physical processes but rather merely to predict the outcomes of experiments. The
measurement problem is that, if the laws of quantum mechanics (the linear laws of state evolution) are taken to characterize experiments (and measurements), then these laws predict that they do not have outcomes. The standard response to this is to exempt measurements from the linear laws and to introduce the projection postulate to calculate the post_measurement state of M+S. From a realist perspective the standard response is problematic. It is difficult to believe that the projection postulate describes a real change of state and, as John Bell (1987) frequently and forcefully emphasized, the occurrence of the notion of measurement (without a purely physical characterization of which interactions are measurements) in the fundamental laws is, if these laws really describe the evolution of real states, unacceptable. We supposed in our original criticisms of the modal interpretation that it was intended as a realist account that is as a true account of physical processes, or at least as providing the basis of true account when extended to relativistic quantum theory. This also seemed to be the attitude of proponents of modal interpretations (except perhaps for van Fraassen). But the baneful influences of previous instrumentalistic attitudes can be seen in Bub's and Healey's defenses. This is all but explicit with Healey, who concludes his paper with the point that non_relativistic quantum theory is false. He then makes the much more modest claim for the modal interpretation that it is capable of describing a possible world in which all measurements have outcomes and a class of those measurements can be called "non_ideal." This is not much of a defense, since there are also non_relativistic worlds in which non_ideal measurements in our sense are common and which possess outcomes but which the modal interpretation fails to assign outcomes. And, as we have argued, it is enormously more likely that our world is one of these worlds.
Footnotes

1. Modal interpretations are those which assign values to observables even when the quantum state is not an eigenstate of the observable in accord with some rule which involves the quantum state or the Hamiltonian that governs the evolution of the state. Healey rejects the name "modal interpretation" for his account preferring "interactive interpretation." In previous papers we cited a number of problems with modal interpretations including the difficulty of adding to them a dynamics for their "hidden variables" and their failure to assign outcomes to non_ideal measurements. It is the latter objection which is at issue in these papers. This objection applies to Kochen's, Dieks' and Healey's interpretations, which all use the polar decomposition theorem to determine which observables possess values, and to Van Fraassen's interpretation which characterizes the observables that possess values in terms of the Hamiltonian governing the interaction. It does not apply to Bub's interpretation which doesn't rely on this theorem.

2. The analysis is slightly more complicated for disturbing measurements. Healey seems to consider it a defect that in our original discussion we restricted our discussion to non_distrubing measurements, but he would surely admit that the added complication is irrelevant to the issues at hand. 3. There are only two honest ways of responding to the measurement problem. One is to break the eigenstate_eigenvalue link. This is the approach taken by modal interpretations, hidden variable theories like Bohm's, and our many_minds interpretation. The other is to modify the laws of evolution of state. This is what is done by orthodox quantum theory, which exempt measurements from the linear laws of state evolution and, more plausibly, by recent collapse theories like GRW, which modify the laws of state evolution so that macroscopic interactions result in states which are close to being eigenstates of familiar observables. 4. When the amplitudes are exactly equal (as in the usual EPR state), there is not a unique bi_orthonormal
form. An advocate of the modal interpretation must argue that since the measure of the set of such states is 0, it is very unlikely that they are actually encountered.

5. For any observable A* there is an initial state that the Hamiltonian evolves into, a state in which A* is well defined. We mentioned this in Albert and Loewer (1991) and owe the point to Yakir Aharonhov.

6. In (1990) and (1991) we showed how the many_minds interpretation and GRW handle inaccurate measurements (viz. interactions that conform to (2). The analysis of these interactions within Bohm's theory is straightforward.

7. The irony here will not be lost on the reader who recalls, from note 5 that Healey must argue that since the measure of the set of states whose bi_orthonormal form has equal amplitudes is 0 one can neglect such states.

8. The Copenhagen interpretation is sometimes so understood; see for example Van Fraassen's (1992) discussion.

References


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Dear Dr. van der Merwe,

Thanks for the referee's report on our paper "non_Ideal Measurements". The second referee is correct that we didn't make it clear that our objection doesn't apply to Bub's theory. We have remedied that in the revision (see bot. p2.). We also fixed up the typos and filled in the references.

Sincerely,

Barry Loewer