To appear in *Philosophy of Science*

David Lewis' Humean Theory of Objective Chance

Many of the most important scientific theories we have contain laws that predict and explain events by specifying their chances. But despite the centrality and importance of chance it is far from clear exactly what chances are and what determines the truth-values of statements that attribute them. In this paper I will be mostly discussing chances assigned by indeterministic dynamical laws of *fundamental* theories; for example certain versions of quantum mechanics. Part I concerns connections between chance, laws, and belief. Part II discusses some accounts of the nature of chance. Part III develops an account of laws and chance due to David Lewis and argues for its superiority over other accounts.

I. The Nature of Chance.

A fundamental theory is one whose basic entities and properties are not constituted or realized by other entities and properties and whose laws do not hold *in virtue of* other laws. It will be useful to have an example of a fundamental theory with indeterministic dynamical laws before us. One such is the version of quantum mechanics thought up by GRW and developed by Bell and Pearle (among others).[1] According to (non-relativistic) GRW the physical state of an isolated system (or the whole world) at time t is specified by its wave function. In GRW it is natural to think of the wave function as a kind of field or "stuff" which occupies a very high (infinite) dimensional space ("configuration" space). Everything else, what particles there are, their positions in 3-dimensional space, the distribution of people, cats and so forth supervenes on the wave function.[2] The basic idea of GRW supervenience is that when the wave function is sharply peaked in a volume of configuration space associate with particle n then n is located in that region.[3] The single dynamical law of GRW is indeterministic. It says that the wave function (of an isolated system or the entire universe) evolves in conformity with a probabilistic law that specifies (depending on the wave function at t) the chances of various wave functions at subsequent times.[4]

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[1] So called "orthodox" or "Copenhagen" quantum theory also contains an indeterministic principle-the projection postulate But those who adopt the Copenhagen account tend to be instrumentalists about quantum mechanics or this part of it. The main reason for this is that the usual formulation of the projection postulate connects it to measurement or observation and it is difficult to take seriously that such notions are part of fundamental laws.

[2] In saying that all such facts "supervene" on the wave function I mean, following Lewis (19xx) and Jackson (1997), that any world that minimally duplicates these facts as well. Whether or not this includes all the facts depends on whether or not GRW is the true and complete physical theory and whether physicalism is true.

[3] For a discussion of how physical properties supervene on the quantum state in GRW see Albert and Loewer (1997).

[4] The law has the consequence that particles that are part of "large" systems of entangled particles (i.e. macroscopic systems) have very narrow wave functions (in configuration space) and so have well defined positions.

More precisely the wave function evolves in accordance with Schrödinger's deterministic law except that at any moment there is a chance of that function collapsing into a narrower – in some of the dimensions of the configuration space- wave function.[5] If a system starts with wave function Ψ the laws specify various possible futures for the system and chances for those futures. I will call chances like these "dynamical objective" chances since they specify how a system evolves and their existence and value is a matter of objective fact.

The best place to begin a philosophical discussion of objective chance is with David Lewis' "A Subjectivist's Guide to Objective Chance". Lewis assumes that objective fundamental chances belong to propositions and in the first instance to propositions that specify that a specific type of event will (or won't) occur at a specific time (or during a particular time interval) in a specific location (or region)). For example, the chance that a GRW wave function collapse centered on point **x** will occur at time t. As Lewis observes the chance of an event A occurring at time t may itself change during the times prior to t so chances are time indexed. At t the chance of A occurring at t+2 may be ½. But at t+1 the chance of A occurring at t+2 may increase to .9. It is assumed that at t the chance of every proposition about times entirely prior to t is either 0 or 1. This assignment of chances gives rise to a tree structure branching toward the future. At each node the branches are the possible futures from that node each weighted by its chance from the time of that node. If at each node there is a chance (or chance density) for every possible subsequent state the chances of each possible future (sequence of states) is determined.

Lewis thinks of chances as being given by what he calls "history to chance conditionals." These are statements of the form "if h is the actual history up to and including t the chance at t of S at t' (t'>t) is x."[6] The totality of history to chance conditionals may be given by a "theory of chance", T. In GRW the chance theory is the fundamental dynamical law. The dynamical chances at t are determined by the state at t; the history prior to t is irrelevant. So the history to chance conditionals are state to chance conditionals of the form "if the state at t is S* then the chance at t of the state at t' being S is x". The chance at t of a proposition R is the sum of the chances of the possible histories branching from the actual history at t at which R is true.

The chances that occur in physical theories like GRW are scientifically significant because of their connections with laws, causation, explanation, and prediction. It will be worth while to briefly spell out some of these connections.

Laws: In a chancy world- one in which events possess fundamental dynamical chances (different from 1 and 0) -events cannot also be governed by deterministic laws. That is, there cannot be a deterministic law that says that S(t) will evolve into S*(t*) while the chance at t of S*(t*) is less than 1. There are some considerations that suggest that

[5] The "measurement problem" doesn't arise in GRW since when a macroscopic system (e.g. a measuring device) becomes entangled with a quantum system (e.g. an electron) the wave function of the composite system is very likely to quickly suffer a collapse to a wave function which is highly peaked in one of the regions corresponding to the device'' pointer positions.

[6] Lewis says that the conditionals are stronger than material conditionals. If the chances are fundamental then it follows from Lewis' account of chances and laws hat these conditionals are nomologically necessary.

fundamental chances *must* be governed by laws. If the state (of an isolated system) at t is identical to its state at t' then although the evolution of the two states may differ it seems that the chances (if there are chances) of the possible evolutions of the two states must be identical. Underlying this is the idea that chances are not *bare fundamental* properties, as, for example, electromagnetic field value might be, but must be grounded in some physical state.[7] So, for example, in GRW (and other indeterministic quantum theories) chances are (nomologically) determined by wave functions.

Causation: In a chancy world chances are part of the causal order; they are caused and are involved in causing. If the laws attribute chances then causation must operate through those chances. What I mean by this is that if C is part of a state that has a chance of resulting in a state of which E is a part then C is a cause of E only by being a cause of E's' chance at some time prior to E. So, for example, if placing a piece of radium near the Geiger counter causes it to register it does so by way of the chance of its registering.[8]

Explanation: In a chancy world an explanation of why one event occurred rather than an alternative may cite the fact that the chance of the first was much greater than the alternative. And in explaining an event we often cite a factor that explains its chance e.g. explaining why the Geiger counter registered by citing the fact that some radium was brought nearby.

Prediction: Chances put limits on the reliability and rationality of our predictions. W cannot reliably do better in predicting whether or not an event E will occur then by being guided by its chance of occurring.

It is hard to see how chances can play their roles in laws, explanation, and causation in the ways outlined above unless they are *objective*; as objective as spatial, temporal, and causal relations. In saying that chances are objective I mean that their existence doesn't depend on our beliefs (except of course where chances of statements about belief are involved.). However, chance is related to belief in a way that can strike one as extraordinary and mysterious. Chance is supposed to *guide* belief. It is not just that our beliefs ought (defeasibly) to track truths about chance; i.e. if the chance of A is x then we ought (other things being equal believe that the chance is x and we ought not believe that the chance is x if its not. In addition to this opinions about the chance of an event rationally constrain our beliefs about that event itself. Lewis proposes a principle which he thinks captures how chance should guide belief he calls "the *Principal Principle"* (PP). The (PP) says, roughly, that a person Ms degree of belief (at t) that A conditional on the proposition that the chance of A (at t) is x should be equal to x; with the qualification that at t she has no information about A that is *inadmissible*. Information about A is inadmissible if it is information about A over and above information about A's chance. In symbols

[7] A way of putting the point is that chances must inhere in the categorical state. One way to understand how a chance is grounded in a categorical state is to say that the chance occurs in a law that specifies the chance of that state evolving into certain other states.

[8] There is a large literature involved with attempting to analyze causation in terms of probability. The relevant notion of probability is often not clearly explained but most accounts seem to appeal to probabilities other than dynamical chances. In any case, none of these accounts is close to being successful.

PP: C(A/P(A)=x&T) = x (where T is any information admissible wrt A)

So, for example, if you believe that the coin in front of you has a chance of ½ of landing heads when flipped then your degree of belief conditional on that and any other information that doesn't tell you anything about the outcome of the flip (except what its chance is) should also be ½. While Lewis initially glosses admissibility as above he later suggests that any information about times prior to the time of A and any laws of nature- including laws that specify chances- are always admissible relative to A.[9]

The PP enables us to infer degrees of belief about frequencies from beliefs about chance- so called "direct inference." So, for example if you think that the chance of an event of type A in circumstances of type E is x and repetitions of E are independent then the PP implies that if there are many repetitions of E your degree of belief that the frequency with which A events occur in E situations is approximately x should be close to 1. Lewis shows how PP fits into Bayesian inference to yield an epistemology for chance propositions. If H and H* are exclusive and exhaustive hypotheses that specify chances for experimental outcomes then since PP says that one's conditional credences (C(e/H), C(e/H*)) should be the chances H and H* respectively assign to e Bayes theorem yields the posterior credences of H and H* on e. The PP is absolutely indispensable to statistical inference.

With this background let's now turn to our main question: *What are the truth makers of propositions about chance?* This is a metaphysical not an epistemological question. The main epistemological questions concerning chance are how do we know the chances of particular events and what justifies believing that there are any chances at all. The PP together with Bayesian inference (or some suitable alternative) will provide an answer to the first question. The second can be answered only once we have answered the metaphysical question. That is, only once we have a view about what chances are.

As we have noted chances are janus-faced. On the one hand they are objective features of reality involved in laws and causation and, on the other hand, they rationally constrain degrees of beliefs via the (PP). We would like an account of chance that explains both of these aspects and, in particular, how one property can possess both kinds of features. That is, how can an *objective* feature of a situation S (the chance of S resulting in event e) which is part of the causal order and is metaphysically distinct from e (it is possible for it to occur without e) *rationally* constrain what degree of belief we should have in e's occurring? That something can possess both features can appear very puzzling; as puzzling as the Mackie found ethical facts that allegedly are part of the causal order and also normatively constrain what we should value. *Prima facie*, objective chances can seem as *queer* as objective values.[10]

II. Accounts of Chance

[9] What is admissible for a given person will in general depend on her credence function. Lewis can be understood as suggesting that rational credence functions are ones in which information about the past and about laws are relevant to a proposition A only by being relevant to the chance of A.

[10] This analogy between chances and values is also made by Black (1999) who thinks it poses a problem for propensity accounts of chance and at least as great a problem for Lewis' account of chance that I discuss later. As the reader will soon see I agree with the first but not the second of these claims.

In this section I examine a number of "interpretations" of probability with an eye to evaluating whether any yields an a notion of chance capable of playing its role as connected with laws, etc and also as guiding belief via the PP.

Probabilities are often identified with frequencies either actual or hypothetical. But fundamental chances cannot be actual frequencies. The first problem is that frequencies apply to *types* of events given *types* of situations but fundamental chances apply to specific events in specific situations. This problem can be overcome, at least for the dynamical chances of fundamental physics, by identifying the chance at t of a particular type of event occurring at t +n with the frequency with which events of that type occur at t +n given a complete (with respect to the fundamental physical theory) specification of the type of situation at t for variable t. But even so the chances of a fundamental theory like GRW cannot be identified with *actual* frequencies as these are almost certainly to diverge from the chances. There are quantum states of isolated systems (or the universe as a whole) which will never occur or occur just once so the frequencies of various outcomes given these states will be either undefined or 1 or 0. And, of course, GRW chances can take irrational values and so cannot be actual frequencies. An adequate account of chance must allow for "frequency tolerance"; the possibility that frequencies diverge arbitrarily far – or almost so-from chances.

Hypothetical frequencies satisfy frequency tolerance (they tolerate probabilities even when the actual frequency is undefined) but face other problems. The hypothetical frequency account identifies the chance of an A type event occurring at t+n when the complete physical situation at t is S with the limit of the frequency with which A occurs at t+n given that the situation at t is S in certain worlds at which S occurs infinitely often. But in which worlds? If in all possible worlds then there is no reason to think that all the worlds in which S occurs infinitely often yield the same- or any- limiting frequency of A. Surely all we care about are nomologically possible worlds. But now we face various problems. First, for fundamental theories like GRW the laws themselves mention chances so we cannot define chances in terms of laws. And, in any case, those laws allow the frequencies even in worlds at which S is repeated infinitely often to diverge (even in the limit) from the chance of A.[11]

There are, of course, connections between chance and frequency but the connections are not sufficiently tight to permit a definition of chance in terms of either actual or hypothetical frequencies. Furthermore, the direction of the connection seems to go from chance to frequencies rather than the other way around. The fact that the chance of an event type E occurring on trial type S is x may be part of an explanations for why a frequency close to x is obtained on many repetitions of S. Also, it is via the (PP) that our beliefs about chances ground our beliefs about frequencies. For example, if the chance of E occurring in situation S is x and S is repeated n times then the PP will tell us what degrees of belief to have in the various possible frequencies of E's occurrence. Of course the PP tells us that the frequency may diverge from the chance.

[11] One suggestion is that the chance of A in situation S is the limit of the frequency of A on S in *most* of the worlds in which S is repeated infinitely many times and at which a limit exists. But what can "most" mean other than "has a probability" of just about 1?

An interesting question is whether frequency accounts- whatever their failings as accounts of fundamental chance- can provide a grounding for the PP. Hypothetical frequency accounts seem incapable of doing so. I want to know what degree of belief I *should* have in the next S being an E. What can the fact that in "most" (relative to some measure) worlds in which S is repeated infinitely often E occurs with limiting frequency x have to do with this belief? Even if "most" (with probability 1) were upgraded to "all" it is hard to see why this fact should rationally constrain my degree of belief concerning the next S. Actual frequency accounts seem better off with respect to the PP. Knowing only that the actual frequency of A on S is x and that occurrence of S are relevantly similar (because they are complete fundamental states) does seem to suggest that one ought to have s degree of belief of x in any particular S being an A. The line of thought is that since there is no relevant difference among the S's one ought to assign the same degree of belief to A occurring in each case and any assignment other than x would be incoherent; that is, someone who assigned a degree of belief other than z to A occurring on each trial and bet at the corresponding odds would be sure to lose But while this argument has some force its first step (the assignment of equal probabilities to each occurrence of A) obviously relies on the principle of indifference.[12] Whether or not the principle of indifference can be formulated free of contradictions and whether or not it is a principle of rationality are controversial matters. But at least we can see that actual frequencies are the right sort of fact to constrain rational degrees of belief in the occurrence of the events whose frequencies they are.

Dissatisfaction with frequency accounts of chance led in the 1950s and 60s to the development of "propensity" accounts of chance. Propensity accounts are minimally committed to one negative and two positive claims. The negative claim is that chances are not metaphysically supervenient on non-chance facts. The latter means that a world that minimally duplicates the non-chance facts of our world also duplicates the facts concerning chances. On the actual frequency account the chance of A on S supervenes on the actual frequency- in fact *is* the actual frequency- of As occurring on Ss. Propensity accounts reject this. The positive claims are 3) the chance that S will result in E is an intrinsic property of the situation S and 4) the chance that S will result in E is a measure of the *propensity* of S to produce E.

The view that chance measures the strength of propensity can be understood in two ways. The first is that propensity is a kind of causation and that causation comes in degrees. The second is that causation doesn't come in degrees but that the chance is a measure of the tendency for S to cause E. I find the first construal very obscure. Propensities are not partial causes in the way, for example, that the road's being slick is a partial cause (together with the driver's talking on her cell-phone, the on coming truck etc.) of the car's running into a tree. Chance doesn't mention the degree to which one factor that is part of a sufficient cause contributes to the effect. On the second construal the causal relation is the same one that is present whenever one event causes another (whether or not chance is involved) but the chance that S will cause E measures the tendency of S to cause E. But what can "tendency" mean here except "chance" and so the account is unilluminating. In any case, on either construal chance play a role in *guiding* the evolution of events. The various chances of S evolving into S1, S2,…are features of S that in some way play a role in connecting S to one of the e's. (Chance, if not quite "the

[12] Howson and Urbach attempt an argument along these lines for a kind of hypothetical frequency account. See Strevens (1999) for an incisive critique of these attempts to justify the PP.

cement of the universe", is either the a weaker adhesive or something that generates cement like connections)

Many philosophers find chances as items that *guide* (sort of metaphysical nudges) the evolution of events to be utterly mysterious. I count myself among them. But that is not the objection to the account that most bothers me That objection is that it is *utterly* mysterious why chances so understood should *guide* rational degrees of belief. David Lewis puts the point forcefully as follows:

"I think there is no refuge here. Be my guest—posit all the primitive un-Humean whatnots you like…But play fair in naming your whatnots. Don't call any alleged feature of reality "chance" unless you've already shown that you something knowledge of which could constrain rational credence. I think I see dimly, but well enough, how knowledge of frequencies and symmetries and best systems could constrain rational credence. I don't begin to see, for instance, how knowledge that two universals stand in a certain special relation N* could constrain rational credence about the future co-instantiations of those universals. (1994 p.484)

Lewis' is referring to Armstrong's account but the point applies generally to propensity accounts. The question is why should a feature of a situation S at t – S's chance of leading to E- have anything at all to do with what degree of belief we should have that E occurs when there is no logical connection between the chance of E and E. As I understand him Lewis is not asking the propensity theorist to provide a demonstration that following the PP will lead to degrees of belief which, when acted on- say as in betting- are guaranteed to be successful. Of course, it is easy to show that the chance of winning bets if one bets in accord with degrees of belief constrained by the PP is high. But concluding from this that it is rational to have a high degree of belief that such action will be successful invokes the PP I doubt that there can be a non-question begging argument that shows that conformity to the PP leads to successful actions.[13] But it is not unreasonable to ask, as I think Lewis is asking, of an account of chance to explain why chances conceived of on that account should have any claim to rationally constrain credence. We saw earlier that actual frequencies can provide a rationale (if not a justification) for the PP although it appealed to the principle of indifference. But it is difficult to see how propensity chance can have anything to do with degrees of belief in propositions that are not about those propensities. The problem is that on the propensity account chances are distinct existences from the events that they are chances of. Worlds that are exactly alike with respect to their occurrent facts may contain different chances. But then why should chances tell us anything about what to believe about what events actually occur? As Lewis says it is completely mysterious why a relations between two universals (degree of propensity or degree of there being a causal relation) should have anything to do with constraining credence about one of those universals.[14]

It is not an option for the propensity theorist to drop the PP. Without it (or a principle which plays the same role and for which the same questions arise) there are no connections between evidence and chances and so no reason to believe any hypothesis about chance. His best course may be to say- echoing a remark of Armstrong's

[13] See Strevens "Objective Probability as a Guide to the World" *Philosophical Studies* 95: 243-275.

[14] Black (19xx) also makes the analogy between the "queerness" of objective values and chance. He persuasively argues that it is very difficult to see how propensity accounts can underwrite the PP but also argues that Lewis' account is no better on this score.

concerning laws- that the PP should be accepted "with natural piety." There may be no account of how propensities can rationally constrain degrees of belief but they do and our constraining degrees of belief by what we take to be the chances has proved to be a successful policy. But I think that this is a desperate response to the problem and would appear even more so if there is a non-propensity account of chance that does provide a rationale for the PP.

One reaction to the problem of providing a rationale for the PP is to give up on the idea that chance is an objective feature of reality and try to make do with just degrees of belief. This heroic course involves attempting to explain why there *appears* to be objective chances in terms of our possessing certain patterns of degrees of belief. Further, the account has to show how the roles of chance in laws, causation, and explanation can be performed even though there are no objective chances. Although this approach has been developed with ingenuity I think it is not likely to succeed. For how can this view account for the chances that occur in a fundamental theory like GRW? It seems it must treat GRW as a compilation of recommendations of the degrees of belief one should have given various situations. But why should one accept these recommendations as opposed to others? If the answer is that GRWs recommendations are, in some sense "better" then the question *what features of objective reality make them better?* If there are such features that *they* will be the truth makers of chance statements.

III. Lewis' Humean Account

In my contribution I want to make a few comments about David Lewis' account of laws and chance and mainly the latter. Lewis develops his account of chance together with his account of laws and as a central part of his program of defending Humean Supervenience (HS). HS is the conjunction of two claims

> 1. The fundamental properties (in addition to spatio-temporal properties and relations) that are instantiated in our world are *local* and *categorical*.
> 2. Any world that duplicates the spatio-temporal distribution of fundamental properties of our world is a duplicate of our world *simpliciter*.

Properties are local if they are instantiated at points and they are categorical if their instantiations metaphysically independent of each other (i.e. no necessary connections). Lewis thinks that it is plausible that the properties inventoried by fundamental physics are local and categorical. I have my doubts about this claim but I think that Lewis' accounts of laws and chance can survive relaxing this requirement. Lewis' account of laws is a sophisticated regularity account which he calls "the Best SysytemAnalysis" (BSA). Let L be a language whose atomic predicates express only fundamental properties (Lewis calls these "perfectly natural properties") in addition to spatio-temporal predicates and mathematics/logic and let W be the set of all truths of L. The truths of L will specify which fundamental properties are instantiated at each point or region. A deductive system in L is a set of sentences of L. The laws are defined as follows:

> Take all deductive systems whose theorems are true. Some are simpler better systematized than others. Some are stronger, more informative than others. These virtues compete: An uninformative system can be very simple, an unsystematized compendium of miscellaneous information can be very informative. The best system is the one that strikes as good a balance as truth will allow between simplicity and strength. How good a balance that is will depend on how kind nature is. A regularity is a law IFF it is a (contingent) theorem of the best system. (1994a p.478)

There may be further conditions that should be put on the law giving systems. For example, physicists seek a system of fundamental laws that cover all physically possible events even if that costs simplicity for little gain in informativeness.

Chances enter the picture by letting deductive systems include sentences that specify the chances of events.

> Consider deductive systems that pertain not only to what happens in history, but also to what the chances are of various outcomes in various situations - for instance the decay probabilities for atoms of various isotopes. Require these systems to be true in what they say about history....Require also that these systems aren't in the business of guessing the outcomes of what, by their own lights, are chance events; they never say that A without also saying that A never had any chance of not coming about. (1995 p.480)

The axiom systems that vie for the title "Best" are formulated in a language whose atomic predicates express natural properties and relations and includes in addition to mathematics and logic a so far uninterpreted function Pt that is thought of as assigning chances to certain propositions at times. All candidates for Best system must be true. The virtues that make for a good system are simplicity, strength, and fit. Simplicity is measure in terms of order of differential equations, number of parameters, length of the conjunction of axioms etc. Strength is measured in terms of informativeness (possibilities characterized in terms of natural predicates excluded). Lewis suggests evaluating "fit" in terms of the likelihood of truths. The higher the chance a system assigns to the true history (or to segments of it given part of the history) the better its fit. So understood fit is a kind of informativeness appropriate for chance. The better a theory fits the facts the more it says about those facts. I will later discuss just how a chance theory provides information. For now let us just observe that these various virtues will typically trade off. Strength and fit can often be improved at the cost of simplicity and visa versa. By assigning chances systems sacrifice strength for fit but may also make great gains in simplicity.

The two main competitors to Lewis' account reject HS. Armstrong agrees with 1 but claims that laws don't supervene on the distribution of natural properties but rather govern the temporal evolution of these properties. Sidney Shoemaker rejects 1 and holds that fundamental properties are individuated at least partially by lawful necessary connections.

So as not to prejudice the issue let me call the laws and chances characterized by the BSA assuming HS "L-laws" and "L-chances." Are they the laws and chances? The answer depends on how well they play the roles of laws and chances in the sciences and how they compare to other candidates for these roles. There are many objections that have been made that seek to show that L-laws and L-chances are not the genuine items. I will consider some and especially a problem about its account of chance below. But before that let us look at some of the attractive features of L-laws and L-chances.

L-laws

1. The account apparently matches (or can be modified in the same spirit) the criteria physicists employ to decide on what the laws are. The system of Newtonian mechanics

was thought to determine the laws (as long as it was thought to be true) because of its simplicity and informativeness. On more metaphysical accounts of law like Armstrong's or Maudlin's the connection between laws and the regularities implied by the best system is not necessary and it is not clear why lawful regularities should turn out to be those implied by the Best System.

> Although the chance function Pt(A) starts off as uninterpreted the Best system account supplies it with truth conditions. Pt(A) is true at w IFF the Best system for w entails Pt(A).
>
> 1. The account applies to token events (or rather to propositions specifying that an event of a particular type will occur at a particular time. A token event is the occurrence of an event type at a time). There is no problem of "the single case."
> 2. The account allows for frequency tolerance – or almost so. Chances can take irrational values and differ from the actual frequencies in repeated situations of the same type.
> 3. The account identifies chance with an objective feature of reality. The account is reductive in that facts characterized without adverting to chance are the truth makers of chance statements. Fourth, the account satisfies HS since worlds exactly alike in their occurrent facts will also be exactly alike in their laws and chances.
> 4. The account connects laws and chance very closely and, in particular, has the consequence that chances are law governed. It also illuminates the preference for dynamical laws (both deterministic and those that specify chances) since such laws apply at arbitrary times and so will typically be very informative.

There are two kinds of objections that are made against HS accounts of laws, chance, etc. One kind involves thought experiments in which we imagine possible worlds that are duplicates with respect to occurrent facts but differ in laws and chances. The other kind involve arguments that claim to show that certain of the jobs that laws or chances are required to do in science cannot be performed by Lewis' versions of these notions or, by any Humean account. An example of the first kind of objection with regard

to chances is that we can imagine two worlds w1 and w2 each of which contain independent repetitions of an experiment of kind E with outcomes y and n but in w1 the chance on each trial of E resulting in y is .5 while in w2 the chance of E resulting in y is .6 and yet the two worlds contain exactly the same sequences of ys and ns. If the worlds contain nothing else then obviously the chances do not supervene on the occurrent facts. I think that the proper response for a Humean to make to these kind of thought experiments is to grant that our ordinary concept of chance allows them and is non-Humean but then to argue a) there is reason to be skeptical about non-Humean laws and chances, b) the features of our nomological concepts that ground the anti-Humean thought experiments are not needed to perform the jobs that science requires of these concepts and are, arguably, a vestige of theological doctrines current when these concepts took shape in the 18th century, and c) that L-laws and L-chances can perform the jobs that science requires of laws and chances.[15] The second kind of objection is more serious. It has been argued that L-laws don't support counterfactuals and don't explain (in whatever they way "genuine" laws are alleged to), that induction would be irrational for someone who believes that the only laws are L-laws (or that it would be irrational or unreliable if the only laws were L-laws), that L-chances do are not connected to explanation (in the way "genuine" chances are alleged to be), and that L-chances not only don't constrain degrees of belief in conformity with the PP but are actually inconsistent with it. If these charges were sustainable then L-laws and L-chances would be poor surrogates for the real items. [16] But I think that none of these charges are correct. Here I will address just the issue of how L-chances fare with respect to the PP.

In "A Subjectivist's Guide…." Lewis claimed that the BSA account of chance is conflicts with the PP. He took this to be very bad news for HS since he also claimed that PP encapsulates everything we know about chance. Lewis called this alleged conflict "the big bad bug" and thought it frustrated his HS program. The problem is that the Best Theory of a world may assign a non-zero L chance at time a time t to a possible future F(t) which is *logically* incompatible with that L-chance. He calls such a future "undermming" since it undermines the theory that assigns it a positive chance. For example, suppose that world w consists entirely of a sequence of 1000 "coin flips" occurring at discrete times about half of which are h and half t so that the Best Theory BT(w) for w says that the flips form a Bernoulli sequences with equal chances of h and t on each flip. Suppose the history to time 500 H(500) consists of an equal number of hs and ts. At time 500 a future F(500) consisting of 500 heads has, according to BT(w), a chance of ½ to the 500 power. F(500) is an undermining future relative to BT(w). By the PP C(F(500)/H(500)BT(w)) = ½ to the 500th power. Let w' be the world composed of H(500)+F(500). BT(w') is certainly different from BT(w). So BT(w) is (supposedly) logically incompatible with w'. But then C(F(500/H(500BT(w)) = 0. Since this is correct the PP is false for L-chance.[17] One can avoid an outright contradiction by claiming that BT(w) is not admissible. This seems not unreasonable since BT(w) provides information about the future. But this is of little comfort according since it points to the fact that in general BT(w) will provide information about the future. If it is never admissible than PP

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I have defended Lewis' account of laws against anti-Humean thought experiments in Loewer 1998.

[16] All of these charges have been made against Lewis' version of Humeanism. But none succeed. For replies to the first two see Loewer (1996) and Loewer (2000).

[17] As we will see this argument is fallacious. One also needs to add to FH the statement that FH is the entire history of the world.

will be useless. Lewis observed that the problem is not specific to the BSA account but will arise on any account of chance compatible with Humean Supervenience on which chances are contingent. Since Lewis thought that the PP encapsulates "all we know about chance" this supposed incompatibility caused him to despair that HS could be defended.[18]

Lewis eventually proposed to solve the problem by replacing the PP with

NP $C(A/T) = P(A/T)$

Since $P(F(500/BT(w)) = 0$ the problem (as given in the example but the point is general) doesn't arise. The NP give almost the same results as the PP for L-chances and exactly the same results on any account of chance on which the true chance theory has chance = 1[19]. Lewis seems content to think that by adopting the NP HS (specifically the Best Theory account) is saved. The fact that L-chances almost satisfy the PP (since the NP and the PP are quite close) allows us to count L-chances as genuine chances, at least as long as there is nothing that plays the chance role any better. A defense of that claim would of course involve showing that propensity chances don't exist.

The apparent need to replace PP by NP may look like an argument against HS and the BSA. But its not. There was no need to trade in the PP for the NP in the first place. Lewis' argument that the PP conflicts with the BSA account is flawed. The mistake is thinking that there are undermining futures. In the argument that F(500) is logically incompatible with T(w) and H(500) we tacitly assumed that w' is completely composed of H(500) and F(500). Without this assumption no contradiction follows. We can see this since H(500), F(500) K and T(w) are (plausibly) compatible where K is the proposition that infinitely many randomly distributed ts and hs occur about half of which are hs. It might be thought that the problem still arises. Let Q be the proposition that the H(500)and F(500) completely describe the world. Then P(F(500)/H(500)&T(w)&Q) is positive (if it is defined) while C(F(500)&T(w)&Q) =0. Contradiction regained! But not so. Q *is*

[18] Propensity accounts of chance don't conflict with PP since chances are logically compatible with any histories. The very feature of propensity accounts that makes them metaphysically and epistemologically suspect (that worlds can completely agree with respect to their non-chance facts but differ in their chances and disagree enormously yet agree on their chances) is what allows them to conform to the PP.

[19] On Lewis' account a chance theory will assign itself a chance less than 1. A propensity account typically will not assign chances to chance statements but it is compatible with those accounts to extend the chance distribution so that the chance theory obtains chance =1.

inadmissable. And since we *never* apply the PP with Q as a condition declaring it inadmissable doesn't render PP useless. So Lewis need never have given up the PP.

Let's return to the question of why we should accept the PP. Recall that Lewis says, somewhat mysteriously, "I can see, dimly but well enough, how knowledge of frequencies and symmetries and best systems could constrain rational credence,…" (Lewis 1994 p484). He seems to be suggesting that on the BSA account chances involve symmetries and frequencies and that these constrain rational degrees of belief. Unlike propensities but like actual frequencies L-chances supervene on non-chance propositions so a principle of rationality that constrains rational degrees of belief with respect to non-chance propositions can entail that chance propositions rational constrain belief. But it is not clear how the principle of indifference can support the PP with respect to L-chance (in fact it seems to conflict with it) and, in any case, the principle of indifference has little to recommend it.[20] But I think that we can see how L-chances can constrain degrees of belief if we recall how they are characterized in terms of Best Theory. Recall that a Best theory is one that best combines simplicity, informativeness, and fit. The fit of a theory is measured by the probability that it assigns to true propositions. Fit can be understood as a kind of *informativeness*- the information that probabilistic propositions provide concerning the propositions they attribute probability to. The higher the probability assigned to true propositions that more informative the theory (the higher the probability to false propositions the less informative and the more misleading the theory). But these probabilities are informative only to someone who is willing to let them constrain her degrees of belief. Now, suppose that someone decides to let the PP constrain her degrees of belief. Then for her the Best theory will be one that best combines simplicity and informativeness- including informativeness as evaluated in terms of the degrees of belief she assigns to true propositions. On this proposal the PP is "built into" the account of L-chance. It can constrain belief because that is part of the account of how a theory earns its title as "Best Theory".

Suppose that instead of adopting the PP someone decides to set her degrees of belief by a different principle; say the anti-PP; C(A)= 1-P(A). When this principle is used to evaluate the informativeness of theories we will count a theory as more informative the lower the probability it assigns to true proposition. Of course the theory that will be counted as Best when using this principle will differ from the Best theory characterized using the PP and the probabilities it defines then will differ (systematically) from the L-chances. Let's call them Anti-L-chances. Anti-L-chances exist just as much as L-chances. The important point is that someone who sets her degrees of belief by the anti-PP applied to anti-L-chances will have degrees of belief that are exactly the same as someone who uses the PP as applied to L-chances. And, of course, there are countless other principles that characterize alternative chance like notions ("chance like" because the characterize functions that satisfy the probability calculus) but which end the end recommend more or less the same degrees of belief (when applied to the chance notion they help characterize) as the PP does for L-chance. Perhaps we could use one of these other principles and one of these other notions of chance. The PP (and L-chance) recommends itself because it is the simplest.

[20] Hoefer provides an example in which the PP applied to L-chance and the principle of indifference conflict and suggests that a Humean should go with the latter.

What we have done so far is provide a rationale for how the PP can sensibly constrain degrees of belief if chances are L-chances. The argument doesn't establish that degrees of belief that fail to conform to the PP for L-chances are incoherent (i.e. violate the probability calculus). There are coherent credence functions that violate the PP for L-chance. Nor does it show that violation of the PP is a violation of some version of the principle of indifference. But someone who violates the PP is, in a fairly straightforward sense, being irrational. On the one hand she accepts that a theory T provides the best combination of simplicity and informativeness and that T recommends that she have a degree of belief of p in a proposition A (as long as she has no inadmissible information wrt A) and yet she opts for a different degree of belief. Such a person is in the position of accepting that a certain person is the best source on what degrees of belief to have regarding certain matters and then opting for different degrees of belief. Notice that this rationale for the PP is not available for propensity chances. Without replying on the PP there is no non-question begging reason to think that setting ones degrees of belief by propensity chances will result in having high degrees of belief in truths and low degrees of belief in falsehoods. And since propositions about propensity chances are facts logically completely distinct from the propositions they assign chances to it is utterly mysterious why they should tell us anything about what degrees of belief to have in those propositions.

> There is a further attractive feature of Lewis' account of chance that I want to briefly mention. Lewis thinks that if the fundamental laws are deterministic then all objective chances must be 0 or 1.
>
> To the question of how chance can be reconciled with determinism....my answer is it can't be done.... There is no chance without chance. If our world is deterministic there is no chance in save chances of zero and one. Likewise if our world somehow contains deterministic enclaves, there are no chances in those enclaves. Postscript to "A Subjectivist's Guide to Objective Chance."
>
> I think that these remarks are certainly correct applied to the propensity account of chance. Propensity chances are essentially *dynamical*; they specify the degree to which one state has a tendency to cause another. If at t the state S deterministically causes S* at t* it can't also be that S* has a dynamical chance of less than 1 of producing S*(t*). But there is no reason to limit L-chances to dynamical chances. And that is a good thing since important theories in physics assign chances to events even though their dynamical laws are thoroughly deterministic. Two examples are statistical mechanics and Bohmian mechanics. Both contain principles or laws that specify probability distributions over initial conditions. There is a tradition of attempting to interpret these probabilities *subjectively*, as degrees of belief. But, for pretty much the reasons mentioned in our earlier discussion of subjective probabilities it is hard to see how subjective probabilities can underwrite the use of probabilities in explanation and laws. For example, within Bohmian mechanics the prohibition on super-luminal signaling follows from the specific Bohmian probability assumption over initial conditions. Other probability distributions permit super-luminal signaling. The failure of

super-luminal signaling seems about as lawful as any generalization. But it would be awkward at best to maintain that its status as *a law is due to our degrees of belief -*to our ignorance*!

It is not difficult to extend L-chances to cover initial condition probability distributions. If by adding such a proposition to a theory one makes a great gain in informativeness with little cost in simplicity than that probability distribution has earned its status as a law and the chances it specifies are as objective as dynamical L-chances. Arguably this is just the case with respect to the micro-canonical distribution in statistical mechanics and the Bohmian probability distribution within Bohmian mechanics. By adding the micro-canonical distribution to Newtonian laws the resulting system (and the proposition that the entropy in the distant past was much lower than currently) entails all of statistical mechanics. By adding the quantum equilibrium distribution ($P(\mathbf{x}) = \Psi^2$) to the Bohmian dynamical laws the resulting system entails standard quantum mechanics. In both cases enormous information gain is achieved with very little cost in simplicity.[21]

Conclusion

Even believers in propensity chances and metaphysical laws should grant (assuming that simplicity and informativeness are clear) that L-chances and L-laws exist.[22] The issue between them and advocates of the BSA account is whether L-laws and chances are the chances and laws that are posited by fundamental theories like GRW and Bohmian Mechanics? Whether or not L-chances are the subject of these theories depends in part on whether L-laws are good candidates for being fundamental laws. If scientific laws can be identified with L-laws then the L-chances they may specify do, I think, play the role of chances in science. They are objective, governed by laws (L-laws), enter into explanations, and underwrite a rationale for the PP. Of course this is a big "IF." The main competition for L-laws and L-chances are from more metaphysical- non-HS views about laws and chance. It must be admitted that these metaphysical accounts fit some of intuitions concerning law and chance better than the L-versions do. In particular the intuition that very different systems of laws and chances can give rise to the same total history of occurrent events is quite strong. If the requirement that anything worthy of the names “law” and “chance satisfy this intuition then HS accounts are dead in the water. But until it can be shown that this requirement is essential if laws and chances are to play the roles that science requires of them then I think we have little reason to give such deference to the intuition. And that is especially so for those who, like myself, find metaphysical laws and chances that are at once metaphysically independent of events and yet govern and their evolution and rationally guide our beliefs utterly mysterious.

[21] See my “Chance and Determinism” (forthcoming) for a development and defense of the idea that L-chances are compatible with determinism.

[22] An interesting question is whether metaphysical laws and propensity chances will match or are likely to match the L versions. It is hard to see how to argue that it is likely that they will match without appealing to the PP but, as I have argued, it is hard to see what justifies the PP for propensity chances.

Barry Loewer
Rutgers
May 4.2000

But that seems to involve some version of the principle of indifference. I know of no plausible statement of that principle which together with the BSA account implies the PP. In any case in it is not obvious that the principle of indifference *rationally* constrains credences. I have a different kind of proposal for providing a rationale for L-chances that follow from the constraints on what makes a theory of the world the Best Theory. As I mentioned earlier there cannot be any non-question begging argument to the conclusion that following the PP will lead - or that one should have a high degree of belief that it will lead- to successful action. So we have to look elsewhere for some kind of a rationale for the PP. The best bet is to find some other principle of rationality that implies the PP. This was basically line of thought that appealed to the principle of indifference to show how certain actual frequencies can constrain credence via the PP. It may be that something like this is what Lewis has in mind.